Assessing Transport in the Ocean and Atmosphere: Computational Tools for Predictability and Experimental Design

Patrick D. Miller

Stevens Institute of Technology
Department of Mathematical Sciences
Castle Point on Hudson
Hoboken, NJ 07030
phone: (201) 216-5452 fax: (201) 216-8321 e-mail: pmiller@stevens-tech.edu
Award Number: N00014-99-1-0029
http://attila.stevens-tech.edu/~pmiller/visual.html

LONG-TERM GOALS

The principal goal in this project is to identify and develop ways in which dynamical systems analysis can contribute to our understanding of predictability in geophysical flows. Secondly, there is a need to combine the numerical tools into a unified computational environment which is both accessible to a wider community of researchers, and easily extended to include additional functionality as new ideas are developed and refined.

OBJECTIVES

The immediate objective is to understand how the geometry of stable and unstable manifolds can be used to address transport questions in flows which are strongly aperiodic in time. An important component of this effort is to identify new ways in which these geometric structures can provide quantitative information regarding transport and mixing, beyond simply quantifying mass transport. Because of the finite-time velocity fields and aperiodic time-dependence, algorithms are needed which help to minimize the effects of such issues as non-uniqeness in the initialization and computation of manifolds, and ambiguities in defining Lagrangian boundaries.

APPROACH

Numerical simulations modeling a wind-forced circulation around an island are being used to investigate ways in which the methods of stable and unstable manifolds can be extended to flows with stronger aperiodic time-dependence. This is joint work with Larry Pratt (WHOI) and Chris Jones (Brown Univ). In the numerical model, an anti-cyclonic circulation is driven by a steady wind stress, setting up a large recirculation gyre to the east of the island. The ratio of the inertial boundary layer thickness, $\delta_I = (U/\beta)^{1/2}$, to the Munk boundary layer thickness, $\delta_M = (A_H/\beta)^{1/3}$, characterizes the importance of relative vorticity advection as compared with dissipation in the western boundary layer. Here U denotes a velocity scale for the flow and A_H is the horizontal eddy viscosity. Increasing δ_I/δ_M increases the degree of time-dependence in the flow. The recirculation attached to the island boundary persists for long times providing an ideal coherent structure for studying Lagrangian transport. As the forcing is increased in these simulations, the lobe geometry becomes less regular in

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **30 SEP 1999**	2. REPORT TYPE	3. DATES COVERED **00-00-1999 to 00-00-1999**

4. TITLE AND SUBTITLE **Assessing Transport in the Ocean and Atmosphere: Computational Tools for Predictability and Experimental Design**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Stevens Institute of Technology,Department of Mathematical Sciences,Castle Point on Hudson,Hoboken,NJ,07030**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR’S ACRONYM(S)
	11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**5**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

both time and space. One challenge is to define a meaningful time-dependent Lagrangian boundary separating the two distinct flow regimes and secondly, to use this boundary to unambiguously quantify the potential vorticity fluxes associated with the lobes.

A separate investigation involves the use of lobe dynamics to quantify mass transport in more complex interactions of stable and unstable manifolds. A single-layer barotropic numerical model is parametrized to produce a solution which evolves into a vortex street. This is the same numerical model used in previous work with Audrey Rogerson (WHOI), L. Pratt and C. Jones, investigating transport in meandering jet structures, however, in the current parameter regime, the core of the jet no longer exists. Paul von Dohlen, a graduate student at Stevens, is working on quantifying the transport between several recirculation gyres (vortices) which interact with one another through heteroclinic intersections of their stable and unstable manifolds.

With regard to the computational tools, the approach is to first overhaul the collection of C-code currently being used in generating the manifolds. The main objectives here are to make the code more extensible, speed up the most intensive computations, and improve the user interface. A second effort is directed toward designing a graphical interface within Matlab which will serve as a model for developing interfaces specific to different elements of the post-processing. Eventually the manifold calculations will also be processed through such an interface. Some of these programming tasks are to be completed by student programmers at Stevens.

WORK COMPLETED

In the simulations of a flow past an island, mass transports have been computed for several levels of wind forcing and the results compared with the transport associated with *Ekman pumping*. In addition, I have investigated and compared several methods for estimating the potential vorticity budget associated with the chaotic advection.

Some preliminary mass transport calculations have been completed for the vortex street derived from the barotropic numerical model.

The package of C-code (*vftool*) went through a significant rewrite over the summer months.

RESULTS

The manifold calculations for the flow past an island show that it is possible to characterize (chaotic) Lagrangian transport even in flows which depart significantly from temporal periodicity. This has now been demonstrated in several simulations where the coherent structures (the recirculation in this case) persist for long times relative to the Lagrangian time scale.

For the simulations of the island circulation, the transport calculations from the lobe dynamics show that even with relatively little time-dependence in the velocity field, the chaotic transport associated with lobes easily dominates the transport associated with Ekman pumping. Figure 1 shows snapshots of the lobe dynamics for four different values of δ_I/δ_M and summarizes the transport calculations.

Progress has been made in developing a method for quantifying the potential vorticity fluxes for the time-dependent Lagrangian structure in the island flows, where the boundary is defined by certain segments of stable and unstable manifolds. At this point the most promising approach appears to be choosing a line segment to connect the stable and unstable manifolds. This segment serves as a “gate” for flow into or out of the particular structure. The idea is modeled after the work of Poje and Haller where it was necessary to define a gyre boundary in the absence of transverse intersections. In general, the definition of the recirculation boundary is ambiguous, depending on choices of intersection points, and the gate is one way to minimize this ambiguity.

IMPACT/APPLICATIONS

These distinguished hyperbolic regions with their associated stable and unstable manifolds are typically robust with respect to perturbations in initial conditions and model parameters. If observational techniques can be developed which make it easier to identify these Lagrangian features in the real ocean, these methods should prove to be of value in helping to validate ocean models.

TRANSITIONS

The software for finding stable and unstable manifolds in numerical vector fields and identifying the lobe dynamics has being used extensively at Brown University. For example, Sean Winkler and Chris Jones are using invariant manifold methods to study transport near the circumpolar vortex in the stratosphere. Drew Poje and George Haller have used the invariant manifold computations to study the dynamics of cross-jet transport associated with transient eddy-shedding events [6].

In the work of Lozier, et al [2], RAFOS float trajectories in the vicinity of the Gulf Stream were viewed in a moving frame. The fate of the trajectories were then interpreted based on the dynamical systems analysis from a numerical model of meandering jets by Rogerson, et al [7].

RELATED PROJECTS

These numerical methods were used to verify certain analytical estimates for the splitting distance between stable and unstable manifolds associated with an exact solution to the viscous, barotropic potential vorticity equation. These computations supported the analysis carried out by S. Balasuriya, C. Jones (Brown), and B. Sandstede (Ohio State) in [1].

George Haller is investigating an approach for identifying hyperbolic structures in flows by evaluating the Jacobian of the vector field along Lagrangian trajectories. This technique may be one approach for systematically identifying distinguished hyperbolic regions and initializing the manifold calculations which serve to organize the mesoscale transport.

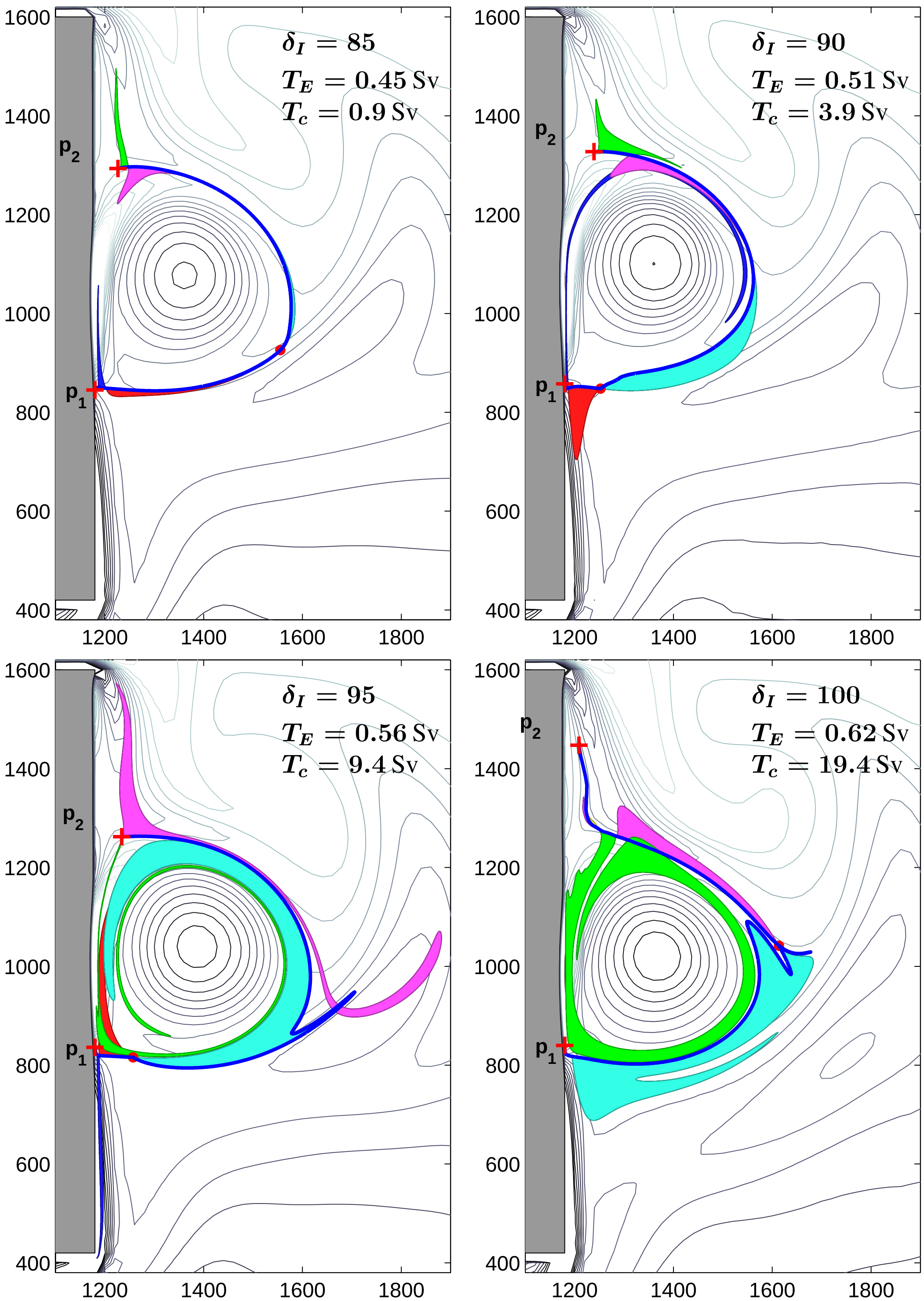

Figure 1. Lagrangian transport for the numerical simulations of flow past an island. $\delta_I = (U/\beta)^{1/2}$ is the inertial boundary thickness, with the Munk layer thickness, $\delta_M = (A_H/\beta)^{1/3}$ fixed at 40 km for all cases. The manifolds are shown at time $t = 50$ days. T_E denotes the *Ekman transport* and T_c the *chaotic transport*.

REFERENCES

[1] H.S. Balasuriya, C.K.R.T. Jones, and B. Sandstede, 1998: Viscous perturbations of vorticity conserving flows and separatrix splitting. *Nonlinearity* 11, 47–77.

[2] M.S. Lozier, L.J. Pratt, A.M. Rogerson, and P.D. Miller, 1997: Exchange geometry revealed by float trajectories in the Gulf Stream. *J. Phys. Oceanogr.*, 27, 2327–2341.

[3] P. Miller, C. Jones, G. Haller, and L. Pratt, 1996: Chaotic mixing across oceanic jets. In *Chaotic, fractal, and nonlinear signal processing, AIP conference proceedings 375,* R. Katz, editor, pages 591–604.

[4] P.D. Miller, C.K.R.T. Jones, A.M. Rogerson, and L.J. Pratt, 1997: Quantifying transport in numerically-generated velocity fields. *Physica D*, 110, 105–122.

[5] J. Pedlosky, L.J. Pratt, M.A. Spall, and K.R. Helfrich, 1997: Circulation around islands and ridges. *J. Marine Res.*, 55, 1199–1251.

[6] A. Poje and G. Haller, 1999: Geometry of cross-stream mixing in a double-gyre ocean model. *J. Phys. Oceanogr.*, 29, 2327–2341.

[7] A. Rogerson, P. Miller, L. Pratt, C. Jones, 1999: Lagrangian motion and fluid exchange in a barotropic meandering jet, *J. Phys. Oceanogr.*, 28, 2635–2655.

[8] V. Rom-Kedar and S. Wiggins, 1990: Transport in two-dimensional maps. *J Arch. Rat. Mech. Anal.*, 109, 239–298.

PUBLICATIONS

A. Rogerson, P. Miller, L. Pratt, C. Jones, 1999: Lagrangian motion and fluid exchange in a barotropic meandering jet, *J. Phys. Oceanogr.*, 28, 2635–2655.